# **Electronic Annex to: Test Validation of Extension-Twisting coupled Laminates with Matched Orthotropic Stiffness.**

**Additional laminate design data and twist augmentation comparisons:**

This supplement provides an additional set of laminate design comparisons to those in the main article.

It was not thought possible that zero *Extension-Shearing* coupled compliances, a16 and a26, could be achieved in the presence of *Bending-Twisting* coupled stiffnesses (see Laminates 5a – 5f), when matching both *Extension-Twisting* (and inseparable *Shearing-Bending*) coupled stiffnesses and orthotropic *Extensional* and *Bending* stiffnesses. However, this was due to an oversight whilst interrogating the laminate databases. Hence new designs are presented here for comparison together to others with matching *Extension-Twisting* (and inseparable *Shearing-Bending*) coupled stiffnesses and orthotropic *Extensional* and *Bending* stiffnesses, as a supplement to the original electronic annex.

**Laminate 4a.**

The stacking sequence for the new baseline comparator, Laminate 4a: [-45/452/-45/90/02/90/45/-452/45/0/902/0]T becomes: [-22.5/67.52/-22.5/-67.5/22.52/-67.5/67.5/-22.52/67.5/22.5/-67.52/22.5]T following off-axis alignment, β = π/8, for which the ABD matrix is given in Eqn. (A1), and represents *E-T-S-B* coupling with isotropic *Extensional Isotropy* as well as *Bending Isotropy*, which are equated by Dij = Aij*H*2/12. The transformed square symmetric relationships for Bij, described in Table 1 of the main article, are also demonstrated.

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|  | (A1) |

The compliance relationship of Eqn. (A2), corresponding to the inverse of Eqn. (A1), reveals that a twist curvature, κxy = -0.00106/mm, arises from using a tensile load Nx = 208.3 N/mm, equivalent to a 5 kN test load.

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|  | (A2) |

**Laminate 5f.**

The stacking sequence for Laminate 5f: [-45/90/45/-45/45/-45/0/45/0/45/90/0/90/0/-45/90]T becomes: [-22.5/-67.5/67.5/-22.5/67.5/-22.5/22.5/67.5/22.5/67.5/-67.5/22.5/-67.5/22.5/-22.5/-67.5]T following off-axis alignment, β = π/8, for which the ABD matrix is given in Eqn. (A3), and represents *E-T-S-B;B-T* coupling with precisely matching stiffness properties to those of Laminate 4a, with the exception that *Bending-Twisting* coupling terms, D16 and D26 are now introduced.

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|  | (A3) |

The compliance matrix of Eqn (4) has zero *Extension-Shearing* coupled compliances, a16 and a26, despite the non-zero *Bending-Twisting* coupling terms, D16 = D26, which would generally result in the compliance matrix becoming fully populated, as is seen in the main manuscript. For the unconstrained deformation, represented by these constitutive relations, only the b16 term influences the magnitude of the twist curvature, κxy = -0.00127/mm, which corresponds to a twist augmentation of 21% above Laminate 4a. Other stiffness matched designs with zero *Extension-Shearing* coupled compliances, a16 and a26, are listed at the end of this section, in ascending order, with respect to twist augmentation above Laminate 4a; as a result of variations in the magnitude of *Bending-Twisting* coupling, D16 = D26.

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|  | (4) |

**Laminate 6r.**

The stacking sequence for Laminate 6r: [-45/453/-454/452/0/902/0/45/-45]T, becomes: [-22.5/67.53/-22.54/67.52/22.5/-67.52/22.5/67.5/-22.5]T following off-axis alignment, β = π/8, for which the ABD matrix is given in Eqn. (5), and represents *E-S;E-T-S-B;B-T* coupling, with precisely matching stiffness properties to those of Laminate 4a, except that *Extension-Shearing* terms, A16 and A26, as well as *Bending-Twisting* coupling terms, D16 and D26 are now introduced.

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|  | (5) |

The compliance matrix of Eqn (6) is fully populated as a result of both the *Extension-Shearing* terms, A16 and A26, and the *Bending-Twisting* coupling terms, D16 and D26. However, for the unconstrained case, only the b16 term once again influences the magnitude of the twist curvature, κxy = -0.00238/mm, which corresponds to an augmentation in the twist curvature of 125% above Laminate 4a.

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|  | (6) |

**Alternative stacking sequences (with off-axis alignment):**

Other stiffness matched designs with are listed at the end of this section, in ascending order of twist augmentation above Laminate 1b.

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| --- | --- | --- | --- | --- | --- |
|  |  | Stacking sequence following off-axis alignment, β = π/8 | D16 | D26 | κxy |
| Laminate 4 | a | [-22.5/67.5/67.5/-22.5/-67.5/22.5/22.5/-67.5/67.5/-22.5/-22.5/67.5/22.5/-67.5/-67.5/22.5]T | 0 | 0 | 0% |
| b | [-22.5/67.5/67.5/-22.5/22.5/-67.5/-67.5/22.5/67.5/-22.5/-22.5/67.5/-67.5/22.5/22.5/-67.5]T | 0 | 0 | 0% |

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| --- | --- | --- | --- | --- | --- |
|  |  | Stacking sequence following off-axis alignment, β = π/8 | D16 | D26 | κxy |
| Laminate 5 | a | [-22.5/67.5/67.5/-22.5/-67.5/22.5/22.5/-67.5/-22.5/67.5/67.5/-22.5/-67.5/22.5/22.5/-67.5]T | -1,518 | -1,518 | 0% |
| b | [-22.5/67.5/67.5/22.5/-67.5/-22.5/-22.5/67.5/22.5/-67.5/-67.5/-22.5/67.5/22.5/22.5/-67.5]T | 3,036 | 3,036 | 1% |
| c | [-22.5/67.5/-22.5/67.5/-67.5/22.5/-67.5/22.5/67.5/-22.5/67.5/-22.5/22.5/-67.5/22.5/-67.5]T | -6,072 | -6,072 | 3% |
| d | [-22.5/67.5/-67.5/-22.5/67.5/22.5/-22.5/67.5/22.5/-67.5/67.5/22.5/-67.5/-22.5/22.5/-67.5]T | -9,108 | -9,108 | 7% |
| e | [-22.5/-22.5/67.5/67.5/-67.5/-67.5/22.5/22.5/67.5/67.5/-22.5/-22.5/22.5/22.5/-67.5/-67.5]T | -12,143 | -12,143 | 12% |
| f | [-22.5/-67.5/67.5/-22.5/67.5/-22.5/22.5/67.5/22.5/67.5/-67.5/22.5/-67.5/22.5/-22.5/-67.5]T | -15,179 | -15,179 | 21% |

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| --- | --- | --- | --- | --- | --- |
|  |  | Stacking sequence following off-axis alignment, β = π/8 | D16 | D26 | κxy |
| Laminate 6 | a | [-22.5/67.5/67.5/-22.5/-22.5/67.5/67.5/-22.5/-67.5/22.5/-22.5/67.5/67.5/-22.5/22.5/-67.5]T | -11,058 | 6,504 | 70% |
| b | [-22.5/67.5/-22.5/67.5/67.5/-22.5/67.5/-22.5/-67.5/22.5/67.5/-22.5/-22.5/67.5/22.5/-67.5]T | -11,058 | 6,504 | 70% |
| c | [-22.5/67.5/22.5/-67.5/-67.5/22.5/67.5/-22.5/-67.5/22.5/-67.5/22.5/22.5/-67.5/22.5/-67.5]T | 6,504 | -11,058 | 70% |
| d | [-22.5/67.5/-67.5/22.5/22.5/-67.5/67.5/-22.5/-67.5/22.5/22.5/-67.5/-67.5/22.5/22.5/-67.5]T | 6,504 | -11,058 | 70% |
| e | [-22.5/67.5/67.5/-22.5/-22.5/67.5/67.5/-22.5/-67.5/-22.5/22.5/67.5/67.5/22.5/-22.5/-67.5]T | -13,589 | 6,000 | 83% |
| f | [-22.5/-22.5/67.5/67.5/67.5/67.5/-22.5/-22.5/-67.5/67.5/22.5/-22.5/-22.5/22.5/67.5/-67.5]T | -13,589 | 6,000 | 83% |
| g | [-22.5/22.5/67.5/-67.5/-67.5/67.5/22.5/-22.5/-67.5/-67.5/22.5/22.5/22.5/22.5/-67.5/-67.5]T | 6,000 | -13,589 | 83% |
| h | [-22.5/-67.5/67.5/22.5/22.5/67.5/-67.5/-22.5/-67.5/22.5/22.5/-67.5/-67.5/22.5/22.5/-67.5]T | 6,000 | -13,589 | 83% |
| i | [-22.5/67.5/-22.5/67.5/67.5/-22.5/67.5/-22.5/-67.5/-22.5/67.5/22.5/22.5/67.5/-22.5/-67.5]T | -14,855 | 5,747 | 90% |
| j | [-22.5/-22.5/67.5/67.5/67.5/67.5/-22.5/-22.5/-67.5/67.5/-22.5/22.5/22.5/-22.5/67.5/-67.5]T | -14,855 | 5,747 | 90% |
| k | [-22.5/22.5/-67.5/67.5/67.5/-67.5/22.5/-22.5/-67.5/-67.5/22.5/22.5/22.5/22.5/-67.5/-67.5]T | 5,747 | -14,855 | 90% |
| l | [-22.5/-67.5/22.5/67.5/67.5/22.5/-67.5/-22.5/-67.5/22.5/-67.5/22.5/22.5/-67.5/22.5/-67.5]T | 5,747 | -14,855 | 90% |
| m | [-22.5/67.5/-22.5/67.5/67.5/-22.5/67.5/-22.5/67.5/22.5/-67.5/-22.5/-22.5/-67.5/22.5/67.5]T | -9,796 | 12,832 | 100% |
| n | [-22.5/-22.5/67.5/67.5/67.5/67.5/-22.5/-22.5/67.5/-67.5/22.5/-22.5/-22.5/22.5/-67.5/67.5]T | -12,832 | 9,796 | 100% |
| o | [-22.5/67.5/67.5/67.5/-22.5/-22.5/-22.5/-22.5/67.5/-67.5/22.5/67.5/67.5/22.5/-67.5/-22.5]T | -12,832 | 9,796 | 100% |
| p | [-22.5/67.5/67.5/-22.5/-22.5/67.5/67.5/-22.5/67.5/22.5/-22.5/-67.5/-67.5/-22.5/22.5/67.5]T | -9,544 | 14,098 | 108% |
| q | [-22.5/-22.5/67.5/67.5/67.5/67.5/-22.5/-22.5/67.5/-67.5/-22.5/22.5/22.5/-22.5/-67.5/67.5]T | -14,098 | 9,544 | 108% |
| r | [-22.5/67.5/67.5/67.5/-22.5/-22.5/-22.5/-22.5/67.5/67.5/22.5/-67.5/-67.5/22.5/67.5/-22.5]T | -12,075 | 13,593 | 125% |
| s | [-22.5/67.5/67.5/-22.5/-22.5/67.5/67.5/-22.5/67.5/-22.5/22.5/-67.5/-67.5/22.5/-22.5/67.5]T | -12,075 | 13,593 | 125% |
| t | [-22.5/67.5/-22.5/67.5/67.5/-22.5/67.5/-22.5/67.5/-22.5/-67.5/22.5/22.5/-67.5/-22.5/67.5]T | -13,593 | 12,075 | 125% |

**Alternative stacking sequences (as manufactured):**

The laminate designs above are more conveniently manufactured with standard alignments and specimens later cut at off-axis alignment, β = π/8.

|  |  |  |
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|  |  | Stacking sequences for ease of manufacture |
| Laminate 4 | a | [-45/45/45/-45/90/0/0/90/45/-45/-45/45/0/90/90/0]T |
| b | [-45/45/45/-45/0/90/90/0/45/-45/-45/45/90/0/0/90]T |

|  |  |  |
| --- | --- | --- |
|  |  | Stacking sequences for ease of manufacture |
| Laminate 5 | a | [-45/45/45/-45/90/0/0/90/-45/45/45/-45/90/0/0/90]T |
| b | [-45/45/45/0/90/-45/-45/45/0/90/90/-45/45/0/0/90]T |
| c | [-45/45/-45/45/90/0/90/0/45/-45/45/-45/0/90/0/90]T |
| d | [-45/45/90/-45/45/0/-45/45/0/90/45/0/90/-45/0/90]T |
| e | [-45/-45/45/45/90/90/0/0/45/45/-45/-45/0/0/90/90]T |
| f | [-45/90/45/-45/45/-45/0/45/0/45/90/0/90/0/-45/90]T |

|  |  |  |
| --- | --- | --- |
|  |  | Stacking sequences for ease of manufacture |
| Laminate 6 | a | [-45/45/45/-45/-45/45/45/-45/90/0/-45/45/45/-45/0/90]T |
| b | [-45/45/-45/45/45/-45/45/-45/90/0/45/-45/-45/45/0/90]T |
| c | [-45/45/0/90/90/0/45/-45/90/0/90/0/0/90/0/90]T |
| d | [-45/45/90/0/0/90/45/-45/90/0/0/90/90/0/0/90]T |
| e | [-45/45/45/-45/-45/45/45/-45/90/-45/0/45/45/0/-45/90]T |
| f | [-45/-45/45/45/45/45/-45/-45/90/45/0/-45/-45/0/45/90]T |
| g | [-45/0/45/90/90/45/0/-45/90/90/0/0/0/0/90/90]T |
| h | [-45/90/45/0/0/45/90/-45/90/0/0/90/90/0/0/90]T |
| i | [-45/45/-45/45/45/-45/45/-45/90/-45/45/0/0/45/-45/90]T |
| j | [-45/-45/45/45/45/45/-45/-45/90/45/-45/0/0/-45/45/90]T |
| k | [-45/0/90/45/45/90/0/-45/90/90/0/0/0/0/90/90]T |
| l | [-45/90/0/45/45/0/90/-45/90/0/90/0/0/90/0/90]T |
| m | [-45/45/-45/45/45/-45/45/-45/45/0/90/-45/-45/90/0/45]T |
| n | [-45/-45/45/45/45/45/-45/-45/45/90/0/-45/-45/0/90/45]T |
| o | [-45/45/45/45/-45/-45/-45/-45/45/90/0/45/45/0/90/-45]T |
| p | [-45/45/45/-45/-45/45/45/-45/45/0/-45/90/90/-45/0/45]T |
| q | [-45/-45/45/45/45/45/-45/-45/45/90/-45/0/0/-45/90/45]T |
| r | [-45/45/45/45/-45/-45/-45/-45/45/45/0/90/90/0/45/-45]T |
| s | [-45/45/45/-45/-45/45/45/-45/45/-45/0/90/90/0/-45/45]T |
| t | [-45/45/-45/45/45/-45/45/-45/45/-45/90/0/0/90/-45/45]T |