Bandwidth and Power Efficiency Analysis of Fading Communication Link

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***Abstract*—Bandwidth and power are the major radio resources of a wireless communication system. Designing an efficient wireless communication system requires trade-off between these resources. In this context, definitions of radio resource efficiencies are given in terms of bandwidth efficiency, bit/s/Hz, and power efficiency, bit/Thermal Noise Energy Unit, for an arbitrary communication system. These efficiencies are analyzed and discussed over Rayleigh and Nakagami fading channels for M-QAM. Investigation shows that the efficiency metrics are degraded by the channel effects. Also, it is evident that each M-QAM level attains highest power efficiency in a specific range of signal to noise ratio (SNR). Thus, an adaptive transmission technique is proposed and analyzed for enhancing resource efficiencies based on number of levels in M-QAM to achieve maximum power efficiency in specific SNR regions. It is shown that efficiency metrics can be significantly improved by using the proposed technique for a broad range of signal to noise ratio, even when fading is severe.**

***Keywords*—Bandwidth efficiency, Power efficiency, Fading channel, M-QAM, Adaptive transmission.**

I. INTRODUCTION

Bandwidth efficiency is an important performance metric for any wireless communication system, and power efficiency is a measure used to tackle the challenges posed by increasing of data traffic and energy consumption. These metrics play significant role in the design of wireless communication systems [1]. Both efficiency metrics must be considered jointly instead of separately. The concept of engaging these efficiencies together in the efficient design of wireless system has been presented and discussed in [2]. Bandwidth efficiency bounds of a wireless channel were defined by Shannon [3] as the maximum achievable transmission rate over a band-limited channel while maintaining an arbitrarily low probability of error. However, for particular modulation technique, bandwidth efficiency is measured as the number of bits that can be reliably transmitted per symbol per unit of bandwidth [4]. Efficient utilization of wireless power resources is not provided by the bandwidth efficiency. Hence, power efficiency definition is introduced to give insight into the power consumption. The power consumption trends in wireless communication systems for the next decade has been introduced and explained in [5] in terms of power per bit rate and power per user. Thus, power efficiency is defined as the maximum bits rate that can be reliably delivered by the wireless communication system per unit of consumed power in the system. Relations and trade-offs between resources efficiencies have been introduced from different perspectives such as wireless link, communication networks and protocols. Four fundamental trade-offs have been introduced in [6] including power-bandwidth trade-off which is our concern in this paper. For the wireless networks, several techniques have been discussed in [7] to improve the power efficiency such as cross layer approach, multiple antennas, cell size reduction and cognitive radio. The trade-off between power and bandwidth in transmission schemes was also explained considering the linearity of power amplifiers. Because the signal to noise ratio (SNR) is a significant measure of the wireless channel and it is directly related to the signal power, it is highly important to link the SNR with power and bandwidth efficiency trade-off. In [8], power efficiency has been defined and analyzed in terms of the number of bits per thermal noise energy unit (TNEU), which corresponds to the number of bits communicated using a signal having the same power spectral density (PSD) as that of AWGN recorded at the receiver. The analysis has shown that the two efficiency metrics contradict with each other and hence a careful study of their trade-off is mandatory for designing future wireless communication systems. For M-QAM modulation technique, the trade-off between bandwidth and power efficiency has been discussed for an AWGN channel in [9] and a technique for enhancing the efficiencies was proposed. For a practical wireless communication system, channel impairments must be included in the analysis of wireless system efficiencies. In this regard, variable-rate and variable-power of adaptive M-QAM over fading channels were studied in [10] with emphasizing to improve the bandwidth efficiency of the system regardless the power efficiency. In recent years, power efficiency in communication systems has gained considerable attention from academia and industry. In this paper, power and bandwidth efficiencies are investigated in different wireless channel models. Trade-off expression of these efficiency indices is also derived in closed form in terms of SNR and channel parameters. In addition, the effect of fading on the power and bandwidth efficiency of M-QAM modulation technique is evaluated as function of channel parameters and constellation size. Improving the efficiency of M-QAM technique based on adapting the constellation size is proposed and analyzed over the fading channel. The remainder of this paper is organized as follows. Wireless channel models and definitions of bandwidth and power efficiencies are presented in Section II. The trade-off between power and bandwidth for a wireless link and M-QAM modulation are analyzed and discussed in Section III. An adaptive M-QAM technique is proposed to enhance the efficiency metrics over a fading channel in Section V. Finally, the work is concluded in Section VI. In this work, bold lower case letter is used to denote a complex variable, e.g., $\boldsymbol{x}$ while x is designated to indicate a scalar variable and the bold math case will be used to denote vectors, $\mathbf{x}$.

II. WIRELESS CHANNEL MODEL

Wireless channel introduces various impairments and effects including small-scale fading, which is due to multi-path, that cause serious degradation of SNR leading to poor wireless communication system performance. Several statistical models are available to describe these effects. The received signal over flat fading can be modeled as

$$y = \alpha\ x + n \tag{1}$$

The received signal over here is modeled as consisting of randomly scaled transmitted signal by a factor α plus AWGN n. The instantaneous SNR and the average SNR can be then described as $\gamma = \alpha^2 E_s/N_o$ and $\overline{\gamma} = \Omega E_s/N_o$, respectively, where $\Omega = \overline{\alpha^2}$ and N_o represents AWGN power spectral density. Many distributions have been derived to model the fading factor, α, of the wireless channel. In tis paper, Nakagami fading channel model is considered and the probability density function (PDF) of α is given as [11]

$$p_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right), \quad \alpha \geqslant 0 \tag{2}$$

and the SNR per symbol, γ, is distributed according to the gamma distribution and is given by

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\overline{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\overline{\gamma}}\right), \qquad \gamma \geqslant 0 \tag{3}$$

The Nakagami channel model is a general distribution and often best fits for wireless multi-path propagation. As the parameter m varies, where $m \in [1/2, +\infty)$, the model spans a wide range of fading environments, including Rayleigh fading ($m = 1$), approximation of Rician ($m > 1$) and AWGN channel ($m = \infty$, no fading).

III. BANDWIDTH AND POWER EFFICIENCY

Power and bandwidth efficiencies are the two main performance metrics for a wireless communication system and how to measure these metrics is crucial in the design of an efficient wireless communication system.

A. *Bandwidth Efficiency*

Bandwidth efficiency quantifies the amount of information bits that can be transmitted over a channel per unit of channel bandwidth. Hence, it characterizes how efficiently the spectrum can be utilized. Shannon [3] specified bounds on bandwidth efficiency for AWGN channel as

$$\eta_{B_b} = \log_2\left(1+\gamma\right), \qquad (bit/s/Hz) \tag{4}$$

However, for a particular modulation technique, the bandwidth efficiency depends on the average mutual information of the modulation technique and its occupied bandwidth. Therefore, it is defined as the number of bits per channel use that can be conveyed over a unit of channel bandwidth and is given by [4]

$$\eta_{B_m} = \frac{I\left(X;Y\right)}{BT_s(M)}, \qquad \left[\frac{bit}{s.Hz}\right] \tag{5}$$

where $BT_s(M)$ (Hz/baud) is the product of occupied bandwidth by the symbol duration assuming no channel interference. For realistic wireless communication system, it is assumed that a practical Nyquist filter with nonzero excess bandwidth (ρ) is constructed at the modulator and demodulator. The excess bandwidth is also called roll-off factor which controls the occupied bandwidth. Thus, $BT_s(M)$ can be related to roll-off factor as $(1+\rho)$. The quantity $I\left(X;Y\right)$ (bits/channel use) is the achievable average information rate of the modulation scheme which can be statistically estimated.

B. Power Efficiency

Power efficiency, η_P, is defined as the maximum amount of information rate (bit/sec) that can be reliably delivered by the wireless communication system per unit of transmission power [12].

$$\eta_P = \frac{R}{P}, \quad (bit/s/Watt) \tag{6}$$

Other definitions of power efficiency have also been introduced which involves the wireless channel effects. Since the SNR is directly linked to the transmitted power, $\gamma = P/BN_o$, power efficiency can be related to SNR as [8]

$$\eta_P = \frac{\eta_B}{\gamma} \tag{7}$$

From the definition of (4) and power efficiency (7), an upper bound on power efficiency for a realistic wireless communication system can be expressed as

$$\eta_{P_b} = \frac{\log_2(1+\gamma)}{\gamma}, \qquad (bit/TNEU) \tag{8}$$

The power efficiency, η_{P_b}, and bandwidth efficiency, η_{B_b}, are plotted as a function of SNR in Fig.1. The sold line in the figure is for the case of AWGN.

C. Power and Bandwidth Efficiency Relationship for Wireless Links

As both efficiencies defined are function of SNR and the SNR is affected by fading over wireless channel, power and bandwidth efficiencies for a wireless link can be obtained by averaging the AWGN efficiency bounds, (4) and (8), over the channel fading distribution, $p_\gamma(\gamma)$, as

$$\eta_{B_f} = \int_0^\infty \log_2(1+\gamma)\, p_\gamma(\gamma)\ d\gamma \tag{9}$$

The upper bound on power efficiency (bit/TNEU) for a given fading channel can be derived using the Khakural's definition [13] and is given as

$$\eta_{P_f} = \frac{\int_0^\infty \log_2(1+\gamma)\, p_\gamma(\gamma)\ d\gamma}{\int_0^\infty \gamma p_\gamma(\gamma)\ d\gamma} \tag{10}$$

For Rayleigh fading channel, the power efficiency bound has been derived in [8] and is given by

$$\eta_{p_R} = \frac{1}{\overline{\gamma}\log 2} e^{\frac{1}{\overline{\gamma}}} Ei\left(\frac{1}{\overline{\gamma}}\right) \tag{11}$$

In this work, the wireless channel is modeled using Nakagami distribution. The bounds on efficiencies can then be obtained in closed form. The bandwidth efficiency can be derived using (3), (9) and [14, eq.(07.34.21.0011.01)] and is given by

$$\eta_{B_N} = \frac{1}{\Gamma(m)\log 2} G_{3,2}^{1,3}\left(\frac{\overline{\gamma}}{m}\ \middle|\ \begin{matrix} 1,1,1-m \\ 1,0 \end{matrix}\right) \tag{12}$$

The power efficiency can be singularly obtained in closed form using (3), (10) and with the aid of [15, 3.383.10, 3.326.2] and is given by:

$$\eta_{p_N} = \frac{e^{\frac{m}{\overline{\gamma}}}}{\log 2}\sum_{k=1}^{m} \frac{\Gamma\left(-m+k, \frac{m}{\overline{\gamma}}\right)(\overline{\gamma})^{k-m-1}}{(m)^{k-m}} \tag{13}$$

where $G[.]$ is the Meijer's G-function and $\Gamma(\beta, x) = \frac{1}{\Gamma(\beta)}\int_x^{+\infty} t^{\beta-1}e^{-t}\ dt$ is the *complementary incomplete gamma function* and m is positive integer. For $m = 1$, (13) reduces to (11). Power and bandwidth efficiencies over Nakagami fading channel are plotted in Fig.1. For

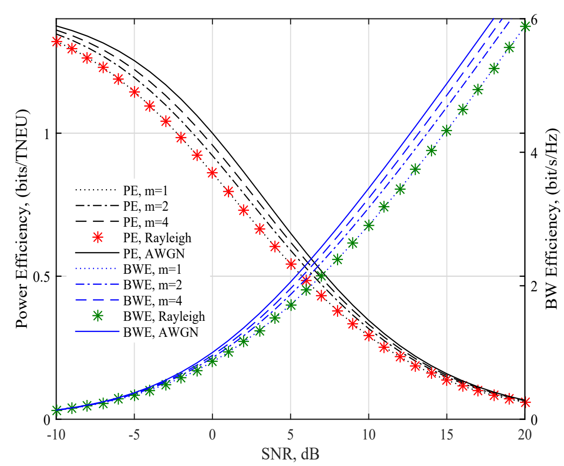

Fig. 1: Power and bandwidth efficiencies for AWGN, Rayleigh and Nakagami as a function of SNR

comparison, boundaries of efficiency metrics are also plotted for AWGN, Rayleigh and several values of m of Nakagami distribution. As can be seen, Rayleigh and Nakagami ($m = 1$) results are the same for both efficiencies. Fig.1 shows that both efficiencies are deteriorated by the channel. For instance, for $m = 1$, the bit/TNEU is reduced by 15% at low SNR values and significantly degraded in high SNR region. In addition, at on SNR of 15 dB, the bandwidth efficiency is decreased by 1 bit/s/Hz for $m = 1$ compared to the case of AWGN. It is also observed that as m increases both metrics approach that for AWGN.

IV. Bandwidth and Power Efficiency of M-QAM

The bandwidth efficiency of M-QAM modulation technique can be defined as the ratio of maximum average information rate to normalized bandwidth as given by (5). The maximum average mutual information rate is estimated by considering the M-QAM modulation system as Discrete-Input Continuous-Output Memoryless Channel (DCMC). Also, the M-ary input symbols are assumed to be equiprobable with a prior probabilities given by

$$p(\mathbf{x}_m) = \frac{1}{M}, \quad m = 1, ..., M. \tag{14}$$

The channel's transition probability for M-QAM is given by

$$p(\mathbf{y}|\mathbf{x}_m) = \prod_{n=1}^{N} \frac{1}{\sqrt{\pi N_o}} \exp\left(\frac{-(y_n - x_{mn})^2}{N_o}\right), \tag{15}$$

where $\mathbf{y}$ is the received signal vector for given $\mathbf{x}$ as the transmitted vector. For $N = 2$ M-ary QAM signaling, the maximum mutual information [16] and hence the bandwidth efficiency using (5) can be written as

$$\eta_{B_M} = \frac{\log_2(M)}{1+\rho} - \frac{1}{M(1+\rho)} \times \sum_{k=1}^{M} E\left\{\log_2\left[\sum_{i=1}^{M} \exp\left(\Phi_i^m\right)\right]\right\} \tag{16}$$

with

$$\Phi_i^m = \frac{-|\boldsymbol{\alpha}(\mathbf{x}_k - \mathbf{x}_i) + \boldsymbol{n}|^2 + |\boldsymbol{n}|^2}{N_o}$$

where $\mathbf{x}_i$ represents the M-QAM symbols and $\boldsymbol{n}$ is a complex AWGN having a variance $N_o/2$ and $\boldsymbol{\alpha}$ is the complex channel gain which can be modeled using a given fading distributions. The expectation $E[.]$ is taken over $\boldsymbol{n}$ and $\boldsymbol{\alpha}$. The power efficiency of M-QAM can be evaluated using (7) where $\gamma = \overline{\alpha}^2|x|^2/N_o$. Bandwidth and power efficiencies are evaluated using Monte Carlo simulation in MATLAB environment. The channels considered are AWGN, Rayleigh and Nakagami. For AWGN the channel gain α is represented by 1 whereas for Rayleigh channel α is simulated as a complex normal random variable. In the case of Nakagami channel, α is expressed by the square root of a gamma random variable. The excess bandwidth occupied is assumed to be 30% of the total bandwidth which is equivalent to $\rho = 0.3$. The simulation is done for one million M-QAM symbols. For AWGN, the bandwidth and power efficiency indexes for M-QAM modulation technique are plotted as a function of constellation size, M, and E_s/N_o in Figs.2 and 3, respectively. Fig.2 shows that the bandwidth efficiency approaches $\frac{\log_2 M}{1+\rho}$ as E_s/N_o increases; however, it is always below the Shannon's limit. On the other hand,

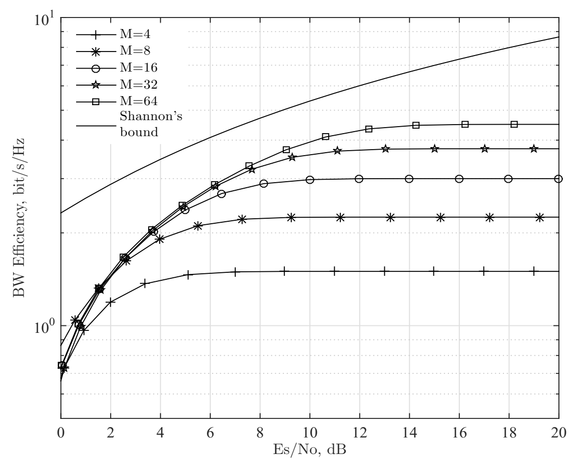

Fig. 2: Bandwidth efficiency of 4, 8, 16, 32 and 64 QAM for AWGN channel

it is observed that the power efficiency deteriorates as E_s/N_o is increased, which indicates enhanced power consumption, as depicted in Fig.3. Also, it is noted that for each modulation level maximum power efficiency is attained in a specific region of E_s/N_o. These regions have well defined boundaries, and are always less than the Shannon's bound. As an example to demonstrate the wireless channel effect on efficiency indexes, the bandwidth and power efficiencies of 16-QAM are plotted for Rayleigh and Nakagami channels in Figs.4 and 5, respectively. It is evident that both metrics are negatively impacted by the wireless channel effects. For instance, in the case of $m = 1$, the power efficiency is degraded by 40% and bandwidth efficiency is reduced by 0.6 bit/s/Hz compared with AWGN. It is noted that the results for Rayleigh and Nakagami ($m = 1$) are the same. From the above analysis and results, it is clear that wireless channel has severely impacted the efficiency metrics. Hence, we need to find an approach that reduces the channel effects. Fig.3 illustrates that the power efficiency achieves a maximum value in specific SNR region for each modulation level. Thus,

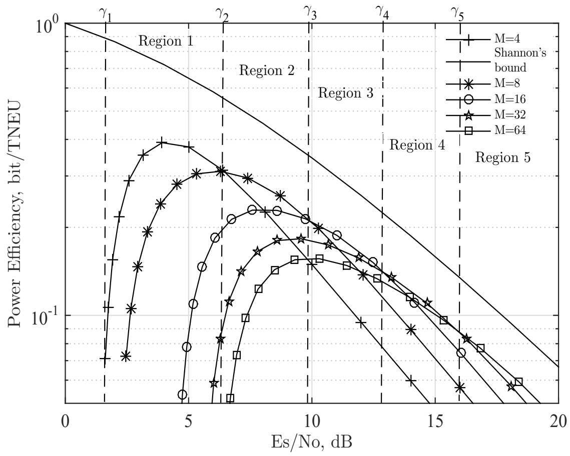

Fig. 3: Power efficiency of 4, 8, 16, 32 and 64 QAM for AWGN channel

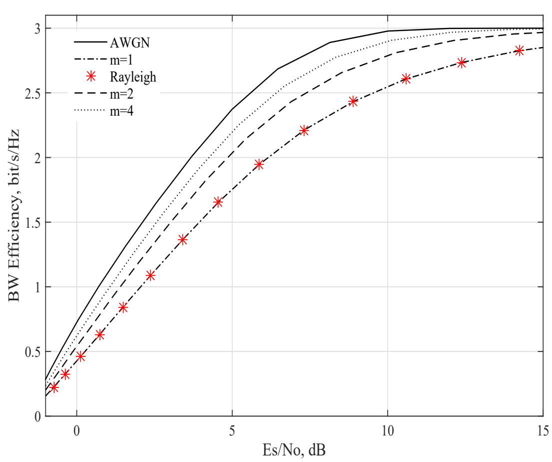

Fig. 4: Bandwidth efficiency of 16-QAM for Rayleigh and different Nakagami

modulation can be adapted to attain optimum power efficiency as a function of SNR estimation.

V. Model of Bandwidth and Power Efficient Technique

Enhancing the bandwidth and power efficiency is our concern in this paper. In the quest for this goal, we propose an efficient power transmission technique that relies on adapting the modulation level, M, to provide optimum power efficiency. In this technique, the SNR

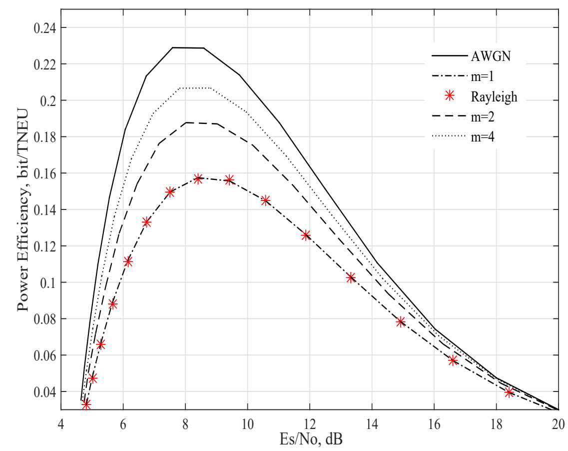

Fig. 5: Power efficiency of 16-QAM for Rayleigh and different Nakagami channels

range is divided into subregions which are specified by boundaries (γ_n and γ_{n+1}). In each region of SNR, a specific M-QAM level achieves maximum power efficiency as shown in Fig.3. For example, 4-QAM provides the highest efficiency in region 1 whereas maximum power efficiency attained by 8-QAM is in region 2. The SNR boundaries are indicated in Table.I which guarantee a certain quality of service. Thus, depending on SNR estimation, an efficient modulation level is determined and then exploited according to Table.I. The constellation size is simultaneously reconfigured at receiver and transmitter. The efficient adaptive system is depicted in Fig.6. The wireless channel is represented as Rayleigh and Nakagami for the analysis of the adapting technique and the channel state information is assumed to be perfectly estimated. The power and bandwidth

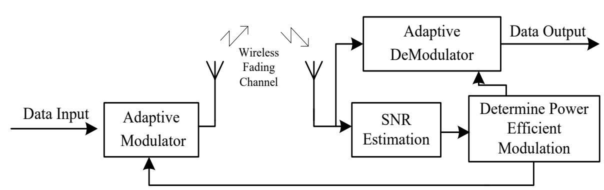

Fig. 6: Block diagram of power efficient adapting technique

efficiencies of the K disjoint regions can be evaluated by first finding the probability that SNR falls in a certain n^{th} region ($P_r(n)$), between γ_n and γ_{n+1}, and then use $P_r(n)$ to obtain the bandwidth efficiency which is equal to the sum of the bandwidth efficiencies, η_{B_M},

TABLE I: SNR BOUNDARIES

Region n	M	γ_n–γ_{n+1} [dB]
1	4	$1.0 - 6.8$
2	8	$6.8 - 9.7$
3	16	$9.7 - 13$
4	32	$13 - 16$
5	64	$16 - \infty$

associated with each region weighted by $P_r(n)$ as

$$\eta_{B_A} = \sum_{n=1}^{K} \eta_{B_M} P_r(n) \quad . \tag{17}$$

The power efficiency is determined using (7) where η_{B_M} is given in (16). The probability that SNR is estimated in a specific region, $P_r(n)$, is derived using

$$P_r(n) = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma)\, d\gamma \tag{18}$$

where $p_\gamma(\gamma)$ is PDF of SNR distribution. For Nakagami fading channel, the $P_r(n)$ is given by

$$P_r(n) = \Gamma\left(m, \frac{m\gamma_n}{\overline{\gamma}}\right) - \Gamma\left(m, \frac{m\gamma_{n+1}}{\overline{\gamma}}\right) \tag{19}$$

and for Rayleigh fading channel, $P_r(n)$ is

$$P_r(n) = \exp\left(-\frac{\gamma_n}{\overline{\gamma}}\right) - \exp\left(-\frac{\gamma_{n+1}}{\overline{\gamma}}\right) \tag{20}$$

The bandwidth efficiency for each M-QAM level η_{B_M} is evaluated using Monte Carlo simulation for the designated regions, and then the overall efficiency metrics are obtained for the proposed efficient technique using (17) and (7). For five SNR regions ($K = 5$), the bandwidth and power efficiencies of the proposed adaptive technique are plotted in Figs.7 and 8, respectively, for Nakagami fading parameters $m = 1, 2, 4$ and Rayleigh fading as a function of SNR. Shannon's bounds are also shown in the plots for comparison. From Fig.7, it is obvious that the adaptive technique performs well to cope the channel effects. Moreover, as SNR increases, the negative effect on the bandwidth efficiency is reduced. Also, Fig.8 shows that the power efficiency is significantly improved for a wide operating range of SNR. However, the adaptive transmission experiences extra system complexity regarding SNR estimation and constellation switching required at the transmitter and receiver as shown in Fig.6. Also, synchronization of constellation size, M, between modulator and demodulator and the impact of feedback delay are other issues in the adaptive transmission technique. Note that Rayleigh fading curve matches the Nakagami curve for $m = 1$. In both figures, it is shown that the efficiencies approach Shannon's bound as m increases.

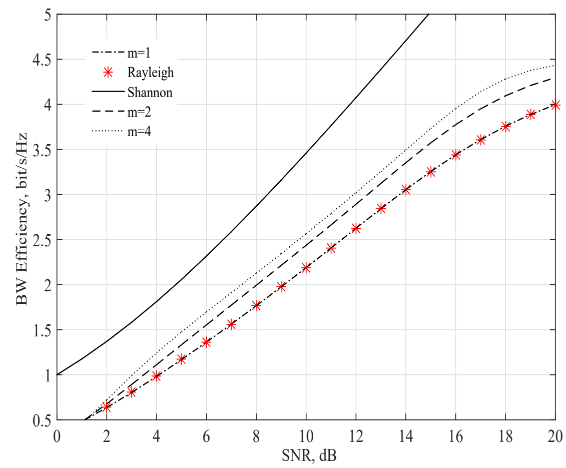

Fig. 7: Bandwidth efficiency of adaptive technique over Rayleigh and Nakagami for several values of m as a function of SNR

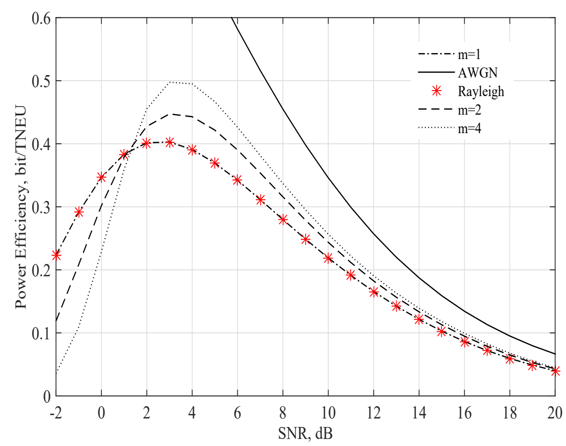

Fig. 8: Power efficiency of adaptive technique over Rayleigh and Nakagami for several values of m as a function of SNR

In the proposed adaptive transmission system, transmission is required to be suspended when the estimated SNR falls below the first boundary, $\gamma_1 = 1$ dB to save energy. Therefore, the adaptive transmission system is subject to an outage probability which can be derived using $P_o = \int_0^{\gamma_1} p_\gamma(\gamma)\, d\gamma$. For Nakagami and Rayleigh channels, P_o are obtained as $P_o = 1 - \Gamma\left(m, \frac{m\gamma_1}{\overline{\gamma}}\right)$

and $P_o = 1 - \exp\left(-\frac{\gamma_1}{\bar{\gamma}}\right)$, respectively. The outage probability for both channels are plotted in Fig.9 as a function of SNR and channel parameters.

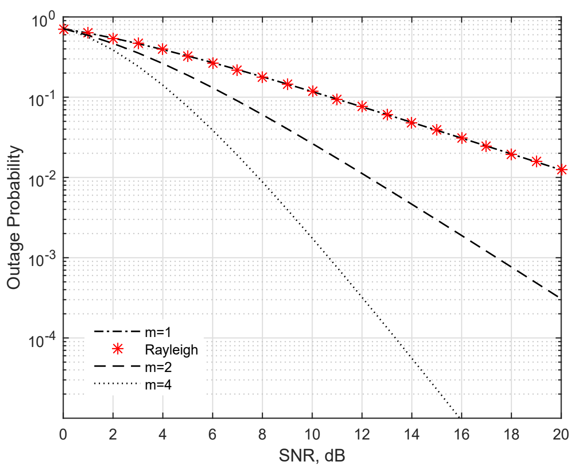

Fig. 9: Outage probability of adaptive efficient transmission technique

VI. CONCLUSIONS

In this paper, the relationship between bandwidth and power efficiency for an M-QAM digital communication system is introduced and discussed. We have investigated these efficiency metrics over Rayleigh and Nakagami fading channels. It is observed that these indexes are degraded by the channel effects and it is possible to improve them by adapting the modulation level as a function of SNR estimation. Adaptive technique for enhancing the efficiency of the system has been proposed and evaluated over several fading channels. The results show that the proposed technique can efficiently ameliorate the overall system efficiency; however, more system complexity is expected.

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